SCM Equations

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January 2023

1 Introduction

At each node i in the soil, we define the normal $\mathbf{n}_i$ and tangential $\mathbf{t}_i$ directions. Compute normal at node from mean of neighhbors. Get tangential from velocity and substract normal part.

At node i we compute the following shear and stresses:

- Bekker-Wong pressure-sinkage

$$\sigma_i = \left(\frac{k_c}{b} + k_\phi\right) z_i^n \tag{1}$$

- Mohr-Columb failure criterion

$$\tau_{max,i} = c + \sigma_i \tan\phi \tag{2}$$

- Janosi-Hanamoto shear stress

$$\tau_i = \tau_{max_i}\left(1 - e^{-J_i/k}\right) \tag{3}$$

Thus, the total force acting on the center of mass of the wheel from the soil node i is given by a normal and a tangential components

$$\begin{aligned} \mathbf{F}_i &= \mathbf{F}_{t,i} + \mathbf{F}_{n,i} && (4)\\ &= A\ (\tau_i\ \mathbf{t}_i + \sigma_i\ \mathbf{n}_i)\,, && (5) \end{aligned}$$

with A the area of the triangular patch.

The total force and torque sums over all soil nodes in contact with the tire:

$$\begin{aligned} \mathbf{F} &= \sum_i \mathbf{F}_i\,, && (6)\\ \mathbf{T} &= \sum_i (\mathbf{r}_i \times \mathbf{F}_i) && (7) \end{aligned}$$