Vehicle Dynamics Modeling

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May 2025

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1 Introduction

In this document, we propose a simplified vehicle dynamics model that captures the key dynamic effects relevant to driving behavior, designed to balance physical accuracy with computational efficiency.

The model essentially describes 2D motion but takes into account road inclination and road bank effects in the computation of forces. It takes into consideration resistance forces such as aerodynamic, rolling and gravitational, as well as the effect of tires friction through the slip angle and cornering stiffness. The drive force is provided through an engine torque curve which depends on the angular velocity of the wheels. The model assumes a set of parameters which is summarized in Table 1.

We consider three dynamic variables: longitudinal and lateral velocities v_x, v_y and yaw rate $\dot{\psi}$ (see Tab. 2). In the following sections dynamic equations for each of these variables are presented. These variables can be integrated to get the longitudinal and lateral displacement and yaw respectively. On the other hand, driver input control is modeled via three quantities: throttle u_t, brake u_b and steering u_δ, see Tab. 3.

The vehicle dynamics presented in this document is mainly based on the modeling from [1, 2, 3, 4].

Symbol	Description	Units
m	Mass of the vehicle	kg
I_z	Inertia moment in the vertical direction	m^{2}kg
l_f, l_r	Front and rear axle positions from the center of mass	m
L_x, L_y, L_z	Vehicle dimensions	m
r_w	Wheel radius	m
C_δ	Steering coefficient	-
T_b	Brake torque	N m
T_e	Engine torque curve	N m
G	Gear drivetrain ratio	-
C_d	Aerodynamic drag coefficient	-
$C_{\alpha f}, C_{\alpha r}$	Cornering stiffness coefficient for front and rear tires	N/rad
μ	Coefficient of friction between tire and road	-

Table 1: Parameters assumed in this model.

Symbol	Description
$\dot{\psi}$	Yaw rate
v_x	Longitudinal velocity
v_y	Lateral velocity

Table 2: Dynamic variables considered in this model.

Symbol	Description	Range
u_t	Throttle	$[0, 1]$
u_b	Brake	$[0, 1]$
u_δ	Steering	$[-1, 1]$

Table 3: Driver input controls.

2 Lateral dynamics

From Newton's second law for motion along the lateral axis, we have [1]

$$m\left(\dot{v}_y + \dot{\psi} v_x\right) = F_{yf} + F_{yr} + F_{bank} \tag{1}$$

where F_{yf} and F_{yr} are the lateral forces at the front and rear wheels respectively, and F_{bank} is the gravitational force due to the road bank angle. On the other hand, moment balance about the z axis yields the equation for the yaw dynamics as

$$I_z \ddot{\psi} = l_f F_{yf} - l_r F_{yr} \tag{2}$$

with l_f and l_r are the front and rear axle distances.

The bank angle force can be written as

$$F_{bank} = mg \sin\phi \tag{3}$$

with the bank angle ϕ defined according to the sign convention shown in right panel of Fig. 2.

Experimental results [1] show that the lateral tire force of a tire is proportional to the slip angle for small slip angles, which is defined as the angle between the orientation of the tire and the orientation of the velocity vector of the wheel θ_{vf} and θ_{vr} for front and rear wheels respectively. Thus, the lateral forces can be written as

$$\begin{aligned} F_{yf} &= 2C_{\alpha f}(\delta - \theta_{vf}) \\ F_{yr} &= -2C_{\alpha r}\theta_{vr} \end{aligned} \tag{4}$$

where δ is the steering angle and $C_{\alpha f}$ and $C_{\alpha r}$ are the cornering stiffness coefficients of front and rear tires, which can be obtained from [5]. θ_{vf} and θ_{vr} can be approximated as

$$\begin{aligned} \theta_{vf} &= \frac{v_y + l_f \dot{\psi}}{v_x} \\ \theta_{vr} &= \frac{v_y - l_r \dot{\psi}}{v_x} \end{aligned} \tag{5}$$

Combining Eqs. 1, 2, 3, 4 and 5 we get the final set of dynamic equations for lateral movement:

$$\dot{v}_y = -\frac{2(C_{\alpha f} + C_{\alpha r})}{m v_x} v_y - \left[v_x + \frac{2(C_{\alpha f} l_f - C_{\alpha r} l_r)}{m v_x}\right]\dot{\psi} + \frac{2C_{\alpha f}}{m}\delta + g\sin\phi \tag{6}$$

$$\ddot{\psi} = -\frac{2(C_{\alpha f} l_f - C_{\alpha r} l_r)}{I_z v_x} v_y - \frac{2(C_{\alpha f} l_f^2 + C_{\alpha r} l_r^2)}{I_z v_x}\dot{\psi} + \frac{2C_{\alpha f} l_f}{I_z}\delta \tag{7}$$

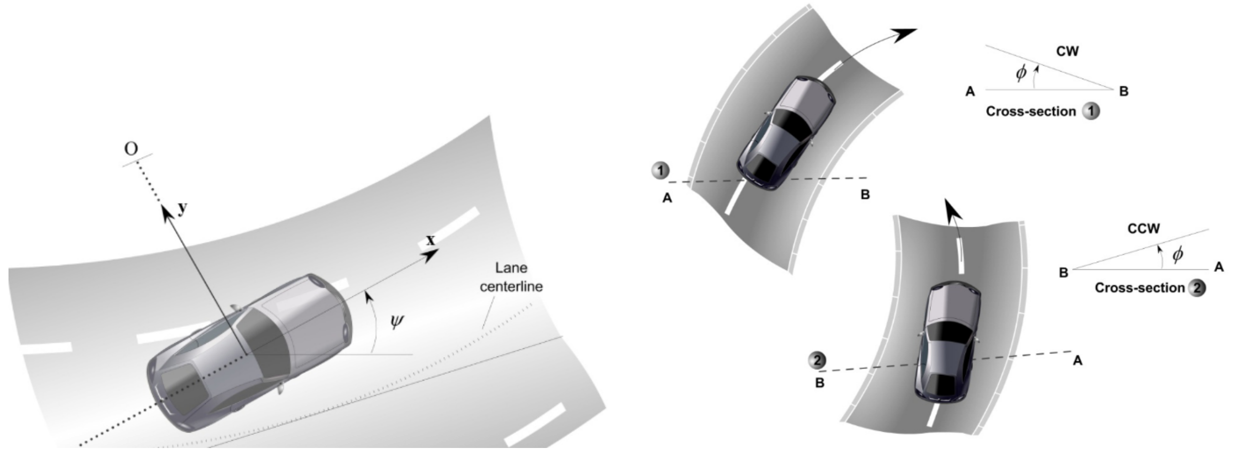

Figure 1: Left: diagram of the lateral dynamics showing the local reference frame and the yaw angle. Right: bank angle ϕ definition convention. Both images extracted from [1].

The steering angle δ can be related to the steering wheel angle input u_δ defined between 0 and 1 as lineraly proportional, $\delta = C_\delta u_\delta$.

Another quantity of interest is the minimum turning radius r_{min} at a given speed. This can be estimated from the fact that at turning, acceleration can be written as $a = v^2/r$. The maximum acceleration will come from the friction component $a_{max} = \mu g$, with μ the dimensionless coefficient of friction between tire and road, and thus r_{min} can be written as

$$r_{min} = \frac{v^2}{\mu g} \tag{8}$$

3 Longitudinal dynamics

From Newton's second law in the longitudinal component of movement we get [2]:

$$m\left(\dot{v}_x - \dot{\psi} v_y\right) = F_{drive} - F_{brake} - F_{load} \tag{9}$$

where F_{drive} is the driving force from the engine, F_{brake} the braking force and F_{load} the total resistance load force. The latter includes aerodynamic resistance F_{aero}, rolling resistance F_{roll} and gravitational force due to road inclination F_g

$$F_{load} = F_{aero} + F_{roll} + F_g \tag{10}$$

The aerodynamic force

$$F_{aero} = \frac{1}{2}\rho A C_d v_x^2 \tag{11}$$

where ρ is the air density, A is the frontal area of the vehicle and C_d is the aerodynamic drag coefficient. The frontal area A is around the 80% of the area calculated from the vehicle width and height for passenger cars [1], i.e. $A \simeq 0.8 L_y L_z$.

The rolling resistance force can be written as proportional to the normal force [3]:

$$F_{roll} = \mu\, mg \cos\alpha \tag{12}$$

with α the inclination angle of the road. The gravitational force due to road inclination is given by

$$F_g = mg \sin\alpha. \tag{13}$$

Lastly, the drive force is proportional to the torque produced in the engine [2]

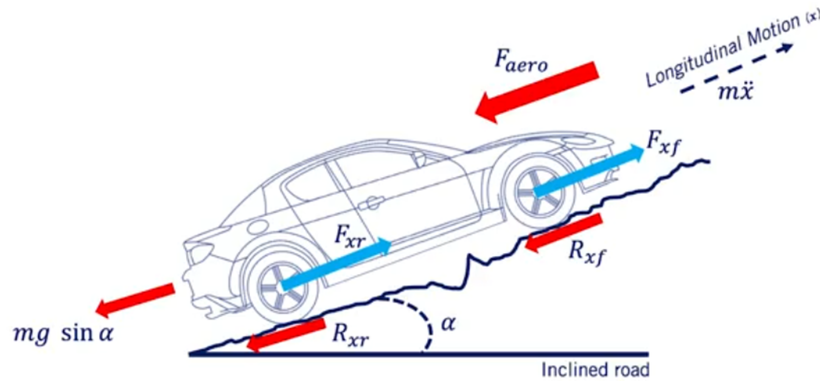

Figure 2: Diagram of the longitudinal dynamics showing the relevant forces, extracted from [4].

$$F_{drive} = u_t \frac{G}{r_{eff}} T_e(\omega_e) \tag{14}$$

where u_t is the throttle input (between 0 and 1), G is the gear drivetrain ratio, r_{eff} the effective wheel radius, which can be approximated to the wheel radius $r_{eff} \simeq r_w$ [1], and $T_e(\omega)$ is the engine torque. This quantity is provided by the torque powertrain curves and is determined by the engine angular velocity ω_e.

The torque curve can be approximated by a second order polynomial, whose coefficients depend on the type of engine [2]. The angular velocity of the engine can be written in terms of the velocity as

$$\omega_e = \frac{G v_x}{r_{eff}} \tag{15}$$

Finally, the braking force can be written as

$$F_{brake} = u_b \frac{T_b}{r_{eff}} \tag{16}$$

where u_b is the brake input (between 0 and 1), and T_b the brake torque. Combining all previous equations, we get a dynamic equation for the longitudinal acceleration:

$$\dot{v}_x = \dot{\psi} v_y + \frac{1}{m}\left(u_t \frac{G}{r_{eff}} T_e(\omega_e) - u_b \frac{T_b}{r_{eff}} - \frac{1}{2}\rho A C_d v_x^2\right) - g\left(\mu\ \cos\alpha + \sin\alpha\right) \tag{17}$$

4 Discussion

In this document we have presented a simplified vehicle dynamic model which aims to capture the essential driving behavior. We discuss in this section some relevant aspects of its implementation.

When integrating the dynamic model into a driving simulator like CARLA [6], all required dynamic information should be accessible from the simulator. That information is in principle available either from the libcarla side or from its python API. We can obtain location and orientation from a CARLA actor through its `actor.get_transform` method. From its roll and pitch, we can derive the road inclination and road bank angles. We can also access velocity and angular velocity of the actor through the `actor.get_velocity` and `actor.get_angular_velocity` methods. The attribute `actor.bounding_box` can be used to get vehicle dimensions. Through the method `actor.get_physics_control` we can get access to other parameters such as the mass, torque curve or gears. Other vehicle variables such as axle positions, wheel radius, brake torques, as well as the drag, friction and cornering stiffness coefficients should be provided independently.

Some simplifications have been assumed through the derivation of the above equations for the sake of simplicity but could be relaxed for more accurate results. For instance, the cornering stiffness behaves linearly only for small slip angles α, i.e. $F_y \simeq C_\alpha \alpha$, as in Eq. 4. But for relatively large angles, a non-linear relation holds, $F_y = f_\alpha(\alpha)$ [5]. Also, the presence of camber angle would have an impact in the estimation of the lateral force [5]. Steering angles are assumed to be the same for the front wheels, but an Ackermann geometry could be considered so that the inner steering angle is lower than the outer one.

Friction is implicitly considered in lateral forces through the cornering stiffness, and thus should be related to the coefficient of friction μ. However, given that the cornering stiffness formulae are empirically set from experiments rather than from first principles, it is non trivial to find a relation between them. This means that there could be configurations of parameters (some specific road conditions for instance) which are not completely consistent given that these parameters are assumed to be independent but are actually correlated.

Finally, it is worth it to mention that this model is an oversimplification of a much more complex problem, reducing all dynamics of a driving vehicle to a single object. In reality, the dynamic system considered is composed by a complex set of components interacting among them with constraints. In order to further expand this proposal to a more accurate 3D model, a dynamic system of particles should be considered, treating each wheel individually, having force and moment equations for each of them, considering suspension and chassis elements and consistently including collisions. This kind of accurate modeling would be more physically accurate at the expense of being more costly computationally, and resembles what is performed in some simulators such as the Chrono simulator [7].

References

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