

Why Dark Matter matters: a brief historical overview

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Although the unknown nature of Dark Matter (DM) has concerned us for decades, it is still one of the most important unsolved problems in modern physics. In order to explain structure formation at both large and small scales, several models of DM have been proposed, composed by different kinds of particles. In this chapter we review the steps which led to the requirement of DM, and the adoption of the Cold Dark Matter (CDM) paradigm. The growth of fluctuations and description of halos are summarized. Motivated by solving small scale problems present in the CDM scenario, several alternative models are reviewed, which lead to the suppression of fluctuations at small scales, discussing their impact on structure formation. We focus on two scenarios: Warm Dark Matter (WDM), such that DM particles have a non-negligible velocity dispersion, free-streaming at low scales; and Interacting Dark Matter (IDM), whose particles interact with photons or neutrinos, damping perturbations by collisional coupling.

1 Why Dark Matter matters

Before discussing specific DM models, we start by overviewing the historical progress of evidences of DM and the consolidation of the CDM paradigm. Some of its possible issues are outlined, motivating alternative DM candidates which are briefly summarized.

1.1 A historical overview

The historical development of the ideas which led to the adoption of the DM as a constituent of the universe has been widely discussed in the literature (see, e.g., Refs. [1, 2, 3, 4, 5].) Although there were hints of the existence of non-visible matter as soon as in the early 20th century (see, e.g., Ref. [1]), the firsts evidences of the existence of such matter were found in the 30s. In 1933 and 1937, Fritz Zwicky made use of the virial theorem with dispersion velocities measured in the Coma cluster, finding the presence of mass that does not emit radiation, about ~ 500 times more than the standard radiative one [6, 7].¹ A similar work was performed by Sinclair Smith in 1936 with data from the Virgo cluster, also finding ~ 100 times more mass than expected [8]. Horace Babcock, in his PhD thesis in 1939, presented the rotation curve of M31 (Andromeda) up to 20 kpc from its center, showing high values for the circular velocity (although he attributed it to a stronger absorption or dynamical effects in the outer parts of the galaxy) [9], and similar findings were drawn from the rotation curve of M33 by Mayall & Aller in 1942 [10]. In 1959, Kahn & Woltjer considered the relative motion between the Milky Way and Andromeda, identifying much more mass than the observed one from stellar origins in order to explain how they are approaching each

¹This ratio is, however, an overestimation of the actual value by a factor of ~ 8 , due to a wrong estimate of the Hubble parameter at that time [1].

other [11]. However, these first hints were not correctly interpreted by the scientific community during several decades.

It was not until the 1970s when strong evidences of the presence of invisible matter were found. Measurements of rotation curves of several galaxies in 21 cm and photometry suggested more mass than expected in the outer regions [12, 13, 14, 15, 16]. In 1973, Ostriker and Peebles performed early numerical N-body simulations, and noted that spiral rotating galaxies were unstable, unless a massive spherical halo were present [17]. Shortly after, two influential papers brought together all the mass discrepancies, evidencing the need for invisible non-baryonic matter, which would be the dominating component, and concluding that the matter density was $\Omega_m \simeq 0.2$, contrarily to the widely assumed value of $\Omega_m = 1$ at that time [18, 19]. A major breakthrough came in 1978 from the rotation curves of a set of galaxies, measured by Bosma in his PhD thesis with the 21 cm line [20], and by Rubin, Thonnard, and Ford in optical observations [21]. Both groups found flat rotation curves well beyond the observed radii of galaxies, meaning that there was invisible mass exceeding the region occupied by stars and gas. At the end of the 70's, the existence of some sort of non-radiating Dark Matter seemed unavoidable [22].

The question then was: which kind of particles compose such invisible matter? Neutrinos seemed to be the perfect candidate for composing such DM, since they had been already measured in experiments, they do not interact with radiation, and the first neutrino oscillation measurements by that time suggested their being massive. The possibility of neutrinos as constituents of the DM was firstly pointed out in 1972 by Cowsik and McClelland [23, 24], and independently by Szalay and Marx in 1976 [25]. Neutrino masses were found to be constrained from cosmological arguments. In 1966, in the first paper considering the role of neutrinos in cosmology, Gershtein and Zeldovich derived an upper bound on the sum of the neutrino masses comparing their energy density with the critical density of the universe around $\lesssim 400$ eV, improving by several orders of magnitude the upper bound in the muonic neutrino mass from earth-based experiments [26].² On the other hand, from the Pauli exclusion principle and assuming neutrino DM as the main constituent of massive halos, Tremaine and Gunn derived a lower bound for the neutrino mass of about $m_\nu > 100$ eV [29] (although it depends on Ω_m). This may be potentially inconsistent with the Gershtein-Zeldovich limit, constraining the range of neutrino masses if they constituted the DM. The announcement (later proven to be wrong) of the detection of an electron antineutrino mass around ~ 30 eV [30] reinforced the possibility of neutrinos as the DM constituent [31], specially in the Moscow's Zeldovich group, who further studied the impact of neutrino *hot* DM (HDM)³ on the growth of fluctuations [33, 34]. HDM was found to present a large free-streaming scale, erasing perturbations below it, and thus providing a *top-down* collapse, where big structures are formed before, and later fragmented to form smaller objects. However, increasingly better N-body numerical simulations during the early 80's contrasted with observations of the CfA, the first 3D galaxy survey [35], ruled out the possibility of neutrino DM, since HDM predicted much less small-scale structures than those observed in data [36].

With light neutrinos not being a plausible candidate, different alternatives were required. Peebles was the first to study the impact on fluctuations of a *cold* DM (CDM), i.e., with negligible free-streaming scale [37]. Contrary to HDM, in a CDM scenario, structure formation proceeds *bottom-up*, presenting power at all scales, and thus forming small-size objects which later merge to

²Cowsik and McClelland re-derived this bound 6 years later [23], being thereafter known as the Cowsik-McClelland limit, despite presenting some mistakes in the computation [27]. The Gershtein-Zeldovich bound with current data is $94 \text{ eV } \Omega_\nu h^2$ [28].

³The terminology distinguishing between Hot, Warm and Cold DM according to the velocity dispersion was proposed in the mid 80's [32].

form larger structures, in a hierarchical way. First simulations of structure formation within the CDM framework resembled the observed clustering properties of galaxies [38], promoting CDM to a promising candidate for the non-visible matter. Collapse of matter lead to the formation of DM halos, whose abundance was well described by analytical estimates of the halo mass function, such as the Press-Schechter formalism [39], or by the Sheth-Thormen prescription, which accounts for the ellipticity of halos [40, 41]. N-body simulations showed that CDM halos have an universal density profile, well fitted by a double power law, now known as the Navarro-Frenk-White (NFW) profile after its authors [42]. This profile, valid over a large range of halo masses, scales as r^{-1} at small radii and as r^{-3} at larger distances, and is completely characterized by its virial mass and radius, and the so-called concentration parameter.

Several particle physics models were able to predict a candidate behaving as this kind of cold, collisionless and non-radiating matter. The prototype of CDM particles are the so-called WIMPs (Weakly Interacting Massive Particles) (term coined in 1985 [43]). These are heavy-mass particles with mass $\gtrsim 1$ GeV in equilibrium with the thermal plasma in the early universe due to weak-like interactions, but decoupling at some moment, freezing out its abundance, which remained mostly constant until now. This mechanism, known as *freeze-out*, allows obtaining the current observed DM density at current times.⁴ The abundance depends mostly on the cross section of the interaction, which is required to be of the order of the weak interactions to produce the observed DM density, coincidence known as the WIMP miracle. Examples of such particles are heavy thermal remnants of annihilation appearing in Supersymmetry, such as neutralinos, the supersymmetric partners of the gauge bosons, which were first considered as DM particles in 1984 [51]. Other popular candidates for CDM are scalar fields, such as axions [52], a hypothetical particle introduced through the so-called Peccei-Quinn mechanism to solve the strong CP problem in quantum chromodynamics [53]. These particles may be produced by non-thermal means, such as from the decay of topological defects or other parent particles. Other popular method is the so-called misalignment mechanism (or vacuum realignment), where the axion field is initially displaced from the vacuum and then relaxes to the potential minimum, behaving as non-relativistic matter [54]. A last group aspirant to constitute CDM, and perhaps the most obvious possibility, are MACHOs (Massive Astrophysical Compact Halo Objects) [55], already suggested during the 70's [56, 57]. With this term, coined by Kim Griest as opposed to WIMPs [1], a variety of objects are englobed which would behave as non-relativistic and non-radiating matter, such as balls of Hydrogen and Helium not massive enough to initiate nuclear burning, like brown dwarfs with masses $\sim 0.01M_{\odot}$ or Jupiter-like planets with masses $\sim 0.001M_{\odot}$. Moreover, black hole remnants from massive stars, or Primordial Black Holes (PBHs) formed in the early universe are also included. Gravitational microlensing is one of the main tools to study them, and has strongly constrained their abundance. However, since MACHOs could only be present in the universe after the formation of first stellar and astrophysical objects, they are unable to successfully explain large scale matter fluctuations seen in the CMB and the number of baryons from BBN. An exception of that are PBHs, which represent a particularly exciting candidate, requiring a special treatment, and will be extensively discussed in Chapter ??.

On the other hand, between the hot and cold limiting cases, an intermediate *warm* scenario was also plausible, with masses around \sim keV which presented a non-negligible free-streaming scale,

⁴The freeze-out of a heavy lepton was independently proposed in five papers published in 1977 during two months by the following groups: Hut [44]; Lee and Weinberg [45] (Lee passing away shortly before the publication); Sato and Kobayashi [46]; Dicus, Kolb, and Teplitz [47]; and Vysotskii, Dolgov, and Zeldovich [48]. However, none of them realized that its relic abundance may be the one needed to constitute the non-visible DM required from astronomical observations [1, 5]. It must be noted, nevertheless, that the freeze-out mechanism, as usually happened in cosmology, had already been studied by Zeldovich and the Moscow group a decade before [49, 50] (see also Ref. [27]).

but still consistent with data and N-body simulations. The first proposals of such WDM particles were gravitinos of mass ~ 1 keV (the spin 3/2 supersymmetric partner of the graviton) in 1982 [58, 59, 60]. Although standard neutrinos were ruled out as DM candidates, other similar species may account for that. It is the case of the right-handed *sterile neutrinos*, non-interacting with Standard Model (SM) particles except by a small mixing with standard active neutrinos. Several mechanisms were suggested to produce them in the early universe from neutrino oscillations, such as the proposed by Dodelson and Widrow in 1993 through oscillations with active neutrinos out of resonance [61], or by Shi and Fuller in 1999 via resonant production [62]. Those particles would have a mass \gtrsim keV, and thus would be a good candidate for WDM (or even CDM). Simulations and data at that time were not accurate enough to discern between the warm and cold scenarios, but CDM started to become the preferred alternative in the community, until becoming the standard cosmological paradigm. However, as shall be reviewed in the following, during the 90's, several problems related to structure formation at small scales challenged the CDM success, revitalizing the WDM alternative.

1.2 Small-scale crisis of the CDM paradigm

The CDM model has shown a great success fitting the data from the large scale structure of the universe. However, there are some discrepancies between observations and N-body simulations at galactic and subgalactic scales, which are not very well explained within the CDM paradigm. Some of these problems arose during the 1990s, when the CDM model predicting hierarchical clustering started to become widely accepted, and N-body simulations improved their resolution to smaller scales. All of them are related to the fact that CDM scenarios predict more small scale fluctuations than those observed in data. Next, we shall review the most relevant issues. See, e.g., Refs. [63, 64] for a comprehensive overview of the subject.

- **Missing satellite issue**

Due to the absence of a cutoff in its power spectrum, CDM models predict a lot of subhalos around massive galaxies. N-body simulations show DM self-bound clumps at all resolved scales, and many more low-mass halos than those present in observations, failing to reproduce the observed circular velocities [65, 66]. Concretely, few dozens of dwarf spheroidal satellite galaxies of the Milky Way have been observed, in contrast to the > 100 satellites present in numerical simulations [67, 68]. The observation of ultra-faint dwarf galaxies by galaxy surveys such as DES have alleviated the problem [69, 70, 71]. Many solutions to this issue have been proposed within the CDM scenario, most of them relying on the fact that not all the subhalos may be visible. Examples of them are based on a suppressed gas accretion in low-mass halos after the EoR [72], or considering supernovae feedback [73], facts which inhibit the formation of stars in small mass halos. Other proposals state that an empirical relation between stellar and halo masses can be used to correct the detection efficiency of galaxy surveys, providing the proper number of counts [74].

- **Cusp-core problem**

A robust prediction from the CDM model which is present in all N-body simulations is the *cuspy* distribution of matter in the inner parts of halos, with density increasing abruptly at small distances from the center. More specifically, CDM density profiles usually rise as $\rho(r) \propto r^{-\gamma}$, with γ between 0.8 and 1.4 over the central radii r [75] ($\gamma \simeq 1$ in the widely used NFW profile [42]). This appears to be in contradiction with the rotation curves of most of the

observed dwarf galaxies, which suggest that they must have flatter central density profiles, i.e., with $\gamma \simeq 0$, coined as *cores* [76, 77, 78]. Hydrodynamic simulations show that it may be possible to settle the problem thanks to baryonic feedback from supernova explosions and stellar winds, which would erase the central cusps [79]. A flat core could also be obtained by considering stellar and gas dynamics, by kinematically heating up DM at the centers of galaxies [80].

- **“Too-big-to-fail” problem**

While the number of low-mass satellites have already been shown to be problematic, the most massive satellite galaxies also present some issues. One naturally would assign the brightest Milky Way (MW) galaxy satellites to the most massive subhalos present in N-body simulations. However, the Aquarius and Via Lactea simulations of the MW showed a population of ~ 10 halos very massive and dense, by a factor of ~ 5 , in such a way that they would be too massive not to host bright dwarf satellites of the MW, which would be more massive than the ones actually observed [81, 68]. This could be understood by the fact that if those very massive subhalos host the brightest satellites, the deep potential wells would lead to circular velocities much larger than the observed dispersion velocity of the observed dwarf galaxies. While in low-mass halos, one can resort to baryonic effects to prevent star formation and then become non-visible, these too massive halos would be too big to fail producing stars and being visible (by baryonic feedback or any other known mechanism), and thus they should be observed. For this reason, this issue is known as the *too-big-to-fail* problem. As in the cusp-core problem, this is related to the fact that CDM tends to produce too much mass in subhalos. Although this issue was originally identified in the MW, it has also been found in the Andromeda satellites [82] and in field galaxies of the Local group, beyond the virial radius of its main galaxies [83]. In order to solve this issue, as well as baryonic feedback, interactions between the MW and its satellites, such as disk shocking or tidal stripping, have been proposed, in order to reduce the central masses of the satellites (e.g., [84]). However, simulations able to properly capture these effects need to resolve very low masses and are very challenging numerically [64].

As already stated, there are several ways to overcome the aforementioned discrepancies within the standard Λ CDM scenario. Baryonic physics, such as stellar winds or supernovae feedback, has been invoked to solve all the above problems, being plausible to account for all of them at once [85]. Other solutions rely on the poorly known mass of the MW, interpreting thus the above issues as possible indicators of a lower mass for the MW than the one assumed [86]. However, DM models different from CDM could also solve some or all of these problems, presenting an interesting and well motivated alternative.

1.3 Non-standard DM candidates

Despite the current efforts on detecting WIMPs, axions, or other possible CDM constituent particles, they remain undiscovered in experiments [87]. This fact, together with the observational discrepancies discussed above, motivate considering other DM models different from the standard cold paradigm. Some examples of these non-CDM candidates, and how they could account for the small-scale discrepancies, are briefly discussed in the following. These models are mostly characterized by their phenomenology in structure formation, rather than by specific particle physics theories. It is worth emphasizing that the term “non-CDM” is employed here to refer to DM scenarios which present different features at small scales, although behaving as CDM at large ones.

With “non-standard”, we also include candidates which can act as CDM with respect to structure formation, but differ from the archetypal WIMP scenario, as is the case of BHs formed in the early universe.

- **Warm Dark Matter (WDM)**

In a typical WDM scenario, DM particles with masses of \sim keV would lead to a substantial velocity dispersion, driving these particles to free-stream and erase fluctuations at small scales. The missing satellite problem is naturally solved, since a cutoff in the power at small scales leads to an underabundance of small structures, compared to the CDM case. As is shown in simulations, WDM can predict the required quantity of subhalos around the most massive ones [88]. Moreover, due to its dispersion velocities, WDM does naturally produce cores. However, to reproduce the observed cores, a WDM mass of \sim 0.1 keV would be required, in a range already ruled out by Ly α forest analyses [89]. Thus, non-ruled out particles with masses \gtrsim 2 keV would not be light enough to satisfy all the current galactic data. Finally, within a WDM scenario with $m_X \sim$ 2 keV, the “too-big-to-fail” issue could be solved due to the relatively shallower profiles of the expected WDM dwarf galaxies compared to their CDM counterparts. However, thermally produced WDM particles with a higher mass particle may not be able to solve the problem satisfactorily [90].

- **Interacting Dark Matter (IDM)**

On the other hand, collisions between DM particles and either photons or neutrinos may avoid the formation of substructures. This happens due to the collisional damping present in IDM scenarios, which, in a similar way to WDM, erase small-scale fluctuations. For this reason, IDM can also explain the low quantity of low mass halos, and thus reconcile expectations with MW satellite observations [91]. Furthermore, as found in high resolution IDM simulations, the largest subhalos are less concentrated than those in the CDM scenario, presenting rotation curves which agree with observations for interaction cross sections of $\sigma \simeq 10^{-9} \sigma_T (\text{GeV}/m_{\text{DM}})$ [92], and thus accounting for the “too-big-to-fail” problem.

- **Self-Interacting Dark Matter (SIDM)**

A widely discussed possibility considers Self-Interacting Dark Matter (SIDM), where, unlike standard collisionless CDM, DM particles present non-negligible interactions among themselves [93, 94]. These collisions would be possibly mediated by hidden gauge fields, and are a generic consequence of those models [95]. Due to scattering, heat would be transferred from high to low velocity particles, enhancing the velocity dispersion of the central regions and reducing the cuspy densities of the halos. For that reason, SIDM was proposed to solve the cusp-core problem [96], which could be explained in this way, as shown in N-body simulations [97, 98]. While the “too-big-to-fail” discrepancy may also be alleviated with SIDM [98], the amount of substructures predicted in simulations is almost identical to that in CDM, and thus the missing satellite problem would remain unsettled [99, 98].

- **Fuzzy Dark Matter (FDM)**

Another popular example considers DM composed by an ultra light scalar field, behaving as axion-like particles (although different from the QCD axion). A specially interesting case is the so-called Fuzzy Dark Matter (FDM), which is a limit of a scalar field DM with masses $\sim 10^{-22}$ eV without self-interactions, behaving as a classical scalar field at cosmological scales [100, 101]. Its evolution is ruled by the Schrödinger equation in the expanding universe, which

can be recasted in continuity and Euler-like fluid equations (the so-called Madelung equations), with an additional effective quantum potential term. This induces an effective Jeans scale (given by a macroscopic de Broglie wavelength), which further suppresses fluctuations at small scales, while FDM behaves as CDM at larger scales [102]. According to these effects, FDM has been proposed to account for the aforementioned small-scale problems [101, 103].

- **Primordial Black Holes (PBHs)**

BHs formed in the early universe from the direct collapse of high density fluctuations conform an interesting candidate for DM, specially after the first measurement of gravitational waves from a merger of BHs by the LIGO collaboration [104]. Regarding its behavior in the formation of structures, these PBHs would mostly act as CDM, although solar mass PBHs may present an enhancement on the fluctuations at small scales due to their discrete distribution [105]. Besides that, it has been claimed that the missing satellite and too-big-to-fail problems may be also alleviated, since the presence of PBHs would imply a large population of ultra-faint dwarf galaxies, in order to be consistent with the LIGO merger rates [106, 107]. Moreover, they would present unique features which may imply different observational effects, such as PBH evaporation or emission of energetic radiation due to accretion. Given the richness of the physics involved, and the variety of phenomenological effects in the evolution of structures and the IGM, Chapter ?? is entirely dedicated to their study.

Driven by solving the above observational issues, these models become plausible candidates for DM. Additionally, some particle physics models may predict such particle candidates, also motivating their study. In this thesis, we focus on studying three of the aforementioned non-standard DM alternatives, WDM, IDM, and PBHs which can leave substantial imprints in the thermal evolution of the universe, the formation of first galaxies, the Reionization epoch and the 21 cm signal. The physical effects, constraints and impact on structure formation of WDM and IDM scenarios shall be discussed along this chapter, while PBHs are studied in the next one.

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